A New LU FACTOR and Interpolation Based DOA Estimation Technique

Yashoda B.S
Research Scholar, Jain University
Bangalore, India.
Yashoda_bs@yahoo.com

Dr. K.R. Nataraj
Professor ECE, SJBIT
Bangalore, India.
nataraj.sjbit@gmail.com

Abstract*: *Smart Antenna increases the capacity of the Mobile Communication System by making use of either Maximal Ratio Combining or Diversity combining techniques. Smart Antenna has to perform a duplex operation. It has to receive signals as well as transmits signals. The reception part mainly requires the detection of user directions. In this paper a novel Signal Subspace, LU Factor method is proposed which detects the users with better bias and Resolution. The algorithm also makes use of interpolation of steering vector without any Hardware resources so that the resolution is improved in a better way.

The LU Factor Method is compared with existing DOA (Direction of arrival) algorithms namely Bartlett, MLM (maximum likelihood method), MEM (Maximum entropy method), MUSIC (Multiple Signal Classification) and QR method. From the Results one can conclude that with respect to bias, resolution and RMSE the LU Factor method is the best.

I. INTRODUCTION

Smart Antenna is a combination of multiple antennas. The smart antenna has 2 major blocks namely Direction of Arrival (DOA) and Beam forming. DOA is responsible for locating the mobile sources by computing the power spectrum while beam forming transmits the radiation in the look direction based on input from DOA. There are many DOA algorithms in the literature each of the approaches have their own way of determining the power spectrum in the network.

II. BACKGROUND

There is a huge amount of work that is performed on the direction of arrival algorithms and this is the latest technology used in mobile communication

The Normalized Power method is an inheritance of Fourier-based spectral analysis [1] to sensor array data. It maximizes the beam for a specific direction.

In the papers [2], [3] statistical based estimation methods such as MLM is derived.

In the paper [4] the antenna array is divided into 2 doublets and then independent eigen vectors will be found on the first L-1 antenna elements covariance matrix and last L-1 covariance matrix. The direction of arrival estimation is performed by using the tangent formula rather than computing the power spectrum.

In the paper [5] source localization is done with planar array for sensors both conditional and unconditional source signal models and in [6] minimal bounds on MSE estimator for general Gaussian observation model is given.

In the paper [7] estimation of quasi-stationary signals is performed and Khatri-Rao (KR) subspace is used to find the DAO in such a way that the noise correlation is reduced but the computation time is very high due to the fact that if other existing DOA methods takes N iterations this methods takes 2N-2 iterations.

In papers [8], [9], [10] expression for Cramer Rao lower bound on the covariance of unbiased estimators of constrained parametric model is derived.

In paper[11] first and second order extended finite impulse response filters are addressed for sub optimal estimation of nonlinear discrete time state space models with AWGN.

MUSIC [12], [13], [14] is an acronym which stands for Multiple Signal Classification. MUSIC provides the estimates of the source directions and then finds out the values in such a way that the bias is less.

In the papers [15], [16], [17] DOA resolution limits in MIMO is discussed.

In paper [18] expressions for partition matrix, inverse matrix and hermitian matrix can be obtained.

In paper [19] sub space tracking algorithms are discussed.

III. Existing DOA ALGORITHMS

A. Normalized Power Method (Bartlett)

In normalized power method first the amplitude matrix is computed and then the steering vectors are computed for all the directions and once they are computed the combination is performed to obtain manifold vector. Once it is obtained then the source correlation matrix and noise correlation matrix are found out and finally the power spectrum is obtained for the variability between -90 degree to +90 degree. The power spectrum for the normalized power method is given by the following equation

$$P_{Normalized\ Power}(\theta) = \frac{S_\theta^H R S_\theta}{L^2} \quad \text{-------- (1)}$$

Where, ‘S_θ’ is steering vector associated with the direction θ, ‘R’ array correlation matrix and ‘L’ antenna elements of equation (1).

B. Maximum Likelihood Method (MLM)

Maximum likelihood method follows the same phenomenon of Normalized Power Method but it computes the inverse of total correlation matrix so that the likelihood is maximized. The power spectrum is computed using the following equation

$$P_{MLM} = \frac{1}{a^H(\theta) R_{inv}\, a(\theta)} \quad \text{------ (2)}$$

Where, $a^H(\theta)$ is the hermitian transpose of $a(\theta)$ and R_{inv} is the inverse of autocorrelation matrix.

C. Maximum Entropy Method (MEM)

MEM DOA method assumes that the entropy is maximized at a time in one specific direction of source. It is build on top of normalized power method and after computation of total correlation matrix it finds the column vector of the correlation matrix which corresponds to maximum entropy and utilizes it in the power spectrum. The power spectrum is given by the equation

$$P_{ME} = \frac{1}{[S_\theta^H C C^H S_\theta]} \quad \text{-------(3)}$$

Where, C is column of R^{-1} and S_θ is the steering vector. $P_{ME}(\theta)$ is based on selecting one of L^{th} array elements as a reference.

D. MUSIC Method (Multiple Signal Classification)

MUSIC method makes use of Noise Subspace in order to find the actual source directions. The Noise Subspace is obtained as the combination of noise Eigen a vector which corresponds to low magnitude. The MUSIC method power spectrum is given by the equation

$$P_{MUSIC} = \frac{1}{a(\theta)^H E_N E_N^H a(\theta)} \quad \text{--------(4)}$$

Where, $a(\theta)$ is steering vector for an angle θ and E_N is L x L-M matrix comprising of noise Eigen vectors.

E. QR Method

This method performs the QR decomposition in order to obtain the power spectrum after finding the noise subspace and then arranges the Eigen values based on the decreasing order. The performance of QR method is similar to that of MUSIC Method.

The array correlation matrix can be defined using the following equation.

$$R = A S A^H + \sigma^2 I -----(5)$$

Where,

$A = Array\ Manifold\ Vector$

$S = Signal\ Correlation\ Matrix$

$A^H = Hermitian\ transpose\ of\ correlation\ matrix$

$\sigma^2 = \text{var}iance\ of\ noise$

$I = Identity\ Matrix$

The QR decomposition is performed for R.

$$R_{QR} = QR(R)$$

Where,

$R_{QR} = QR\ Decomposition$

$equivalent\ of\ corelation\ matrix$

For the QR decomposed matrix Noise Eigen vectors are found out and substituted in the power spectrum equation.

F. LUI Method

LU Factor and Interpolation Method finds the Eigen subspace and then find the LU Factor and determine the directions and then finally substitute the value in the power spectrum equation.

Notation	Meaning
N_a	Number of Antenna
N_M	Number of Mobile Users
d	Distance between antenna elements
S_v	Steering Vector for an angle
M_{AM}	Manifold vector for multiple steering vectors
A_v	Amplitude vector
σ^2	Variance of Noise
Sub_{Noise}	Noise Subspace
Sub_{signal}	Signal Subspace
ev_i	Eigen vector for i^{th} eigen value
LU_f	LU Factorization
S_I	Interpolated Steering Vector
$P_{spectrum}$	Power Spectrum for LU Factor
λ	Wavelength
θ_i	i^{th} *angle*
TC	Total Correlation
C_{noise}	Noise Correlation

Step1: The LUI Method first computes the Interpolated manifold vector which provides the delay computation of em waves for various angles

$$M_{AM} = \begin{bmatrix} 1 & 1 & 1 \\ e^{-j\frac{2\pi}{\lambda}d\sin(\theta_1)} & e^{-j\frac{2\pi}{\lambda}d\sin(\theta_2)} & e^{-j\frac{2\pi}{\lambda}d\sin(\theta_{NU})} \\ e^{-j\frac{6\pi}{\lambda}d\sin(\theta_1)} & e^{-j\frac{6\pi}{\lambda}d\sin(\theta_2)} & e^{-j\frac{6\pi}{\lambda}d\sin(\theta_{Ni})} \\ . & & \\ e^{-j(N_I-1)\frac{2\pi}{\lambda}d\sin(\theta_1)} & e^{-j(N_I-1)\frac{2\pi}{\lambda}d\sin(\theta_1)} & e^{-j(N_I-1)\frac{2\pi}{\lambda}d\sin(\theta_{Ni})} \end{bmatrix}$$

Step2: Compute the hermitian transpose of the Interpolated Manifold vector. $M^H{}_{AM}$

Step3: The amplitude vector for N_i number of users is computed using the following equation with the assumption of cross correlation is zero.

$$A_v = \begin{bmatrix} a_1 a^*_1 & & & 0 \\ 0 & a_2 a_2^* & \ldots\ldots\ldots\ldots & 0 \\ & & & \\ 0 & 0 & \ldots\ldots\ldots\ldots & a_{N_M} a^*_{N_M} \end{bmatrix}$$

Where,

$a_i = i^{th}$ *amplitude value*

Step4: The total correlation matrix is computed using the following equation

$$TC = M_{AM}\, A_v\, M^H{}_{AM} + C_{noise} \text{ --------- } (6)$$

Step5: The Eigen values are found out for the total correlation matrix and then the Eigen values are grouped which are having high dimension. The roots of the following equation are found to get Eigen values

$$|TC - \lambda I| = 0 \text{ --------------------------------}(7)$$

Step 6: The signal subspace for the Eigen values is found out for a set of higher magnitude Eigen values.

Consider a set $\{\lambda_1, \lambda_2, \ldots\ldots\ldots\ldots, \lambda_{N_M}\}$ which are signal Eigen values then Eigen vectors are found out for all N_M.

Let Sub_{signal} represent the signal subspace which combines the Eigen vectors for all the Eigen values

Step 7: Perform the LU Factorization for Signal Subspace LU_f

Step 8: The power spectrum is then computed using the following equation .

$$P_{spectrum} = \frac{1}{S^H{}_\theta LU_f \, LU_f^H S_\theta} ----(8)$$

Where,

$S = steering\ vector\ \ for\ angle\ \theta$

$S^H = hermitian\ transpose$

$LU_f = LU\ \ Factorization$

$LU_f^H = hermitian\ transpose\ \ for\ LU\ \ factorization$

IV. Simulation Results:

Set Up

Parameter Name	Parameter Value
Type of Antenna	Dipole
Type of Array	Uniform Linear Array
Variability	$-90^0 \le \theta \le 90^0$
Antenna Separation	$\frac{\lambda}{2}$

Parameters for Simulation:

1. **Resolution:** The capability of an algorithm to distinguish between equal energy sources with nearly equal angles

2. **Bias:** The bias is computed using the following equation .

$$B = \left|\theta_{true} - \theta_{actual}\right| ----(9)$$

Where,

$\theta_{true} = true\ direction$

$\theta_{actual} = actual\ direction$

If the bias is less then algorithm is good.

3. **RMSE:** The RMSE error is found by taking the values of bias for various variations of angles.

$$RMSE = \sqrt{\left|\theta_{true} - \theta_{actual}\right|^2} ---(10)$$

$$\theta_{initial} \le \theta + 5^0 \le \theta_{end}$$

Where,

$\theta_{initial} = initial\ \ value\ of\ \ direction$

$\theta_{end} = end\ \ value\ of\ \ direction$

Resolution Comparison:

Case1: Low RF Elements and Far Away Users

Parameter Name	Parameter Value
Number of Antenna Elements	8
Number of Users	3
Amplitude of Sources in volts	[1v 2v 3v]
Direction of Sources	[30 45 60]

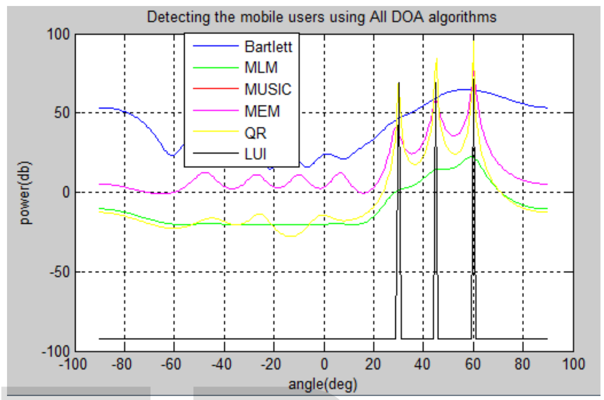

Fig1: Performance Analysis 1

Fig1 shows the Performance Analysis1 as shown in the figure the proposed LUI Method, QR, MUSIC and MEM perform better as compared to MLM and Bartlett.

Case2: Low RF Elements and Nearby Users

Parameter Name	Parameter Value
Number of Antenna Elements	8
Number of Users	3
Amplitude of Sources	[1v,2v ,3v]
Direction of Sources	[30 34 38]

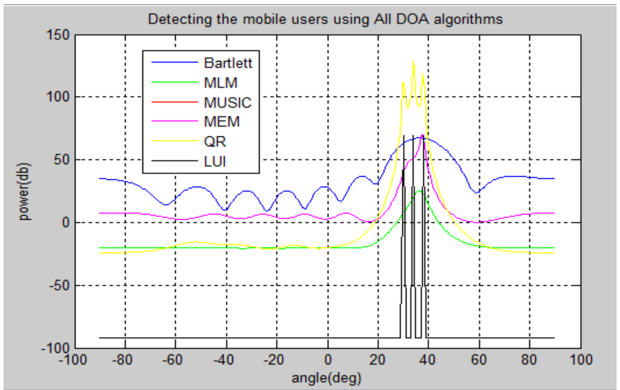

Fig2: Performance Analysis 2

Fig2 shows the Performance Analysis2 as shown in the figure LUI, QR, MUSIC performs the best as compared to MLM and Bartlett.

Case3: Large RF Elements and Far Away Users

Parameter Name	**Parameter Value**
Number of Antenna Elements	100
Number of Users	3
Amplitude of Sources	[1v 2v 3v]
Direction of Sources	[30 45 60]

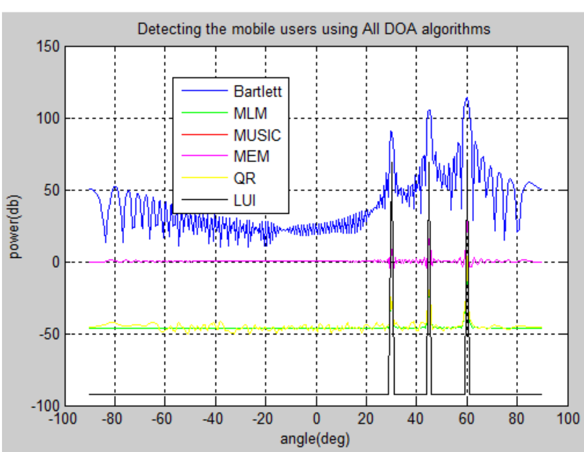

Fig 3: Performance Analysis 3

Fig3 shows the Performance Analysis3 as shown in the figure all algorithms perform better but the sharpness of LUI method is the best

Case4: Large RF Elements and Nearby Users

Parameter Name	**Parameter Value**
Number of Antenna Elements	100
Number of Users	3
Amplitude of Sources in volts	[1v,2v, 3v]
Direction of Sources	[10 13 16]

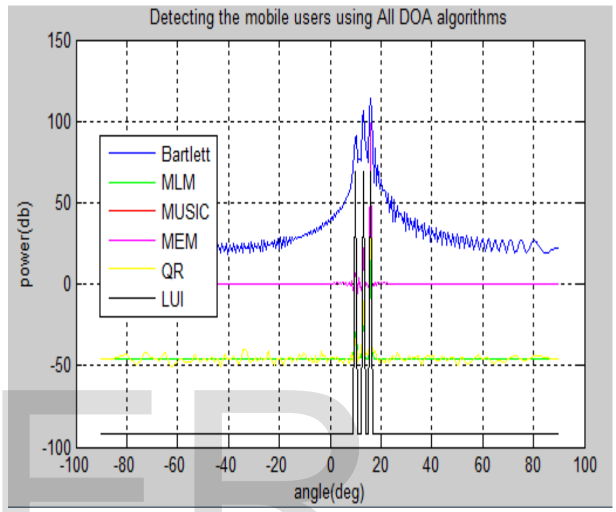

Fig4: Performance Analysis 4

Fig4 shows the Performance Analysis4 as shown in the fig all algorithms perform better.

Bias Comparison :

Parameter Name	**Parameter Value**
Number of Antenna Elements	8
Number of Users	1
Amplitude of Sources in volts	1v
Direction of Sources	45

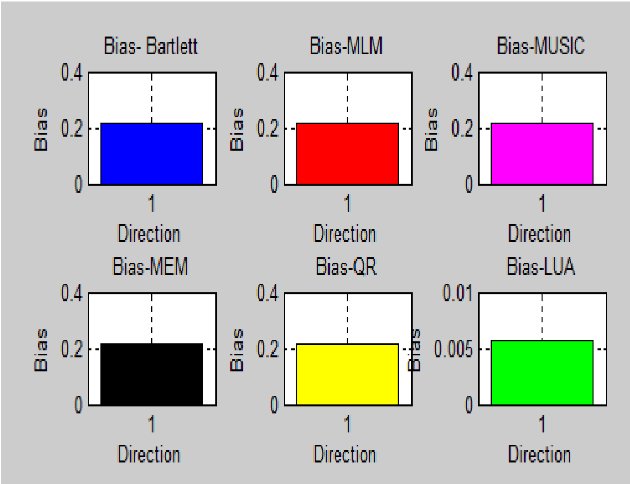

Fig5: Bias Computation

Fig5 shows the bias computation as shown in figure the proposed LUI method has lowest bias of 0.005 as compared to remaining methods namely Bartlett, MLM, MEM, MUSIC and QR have bias closer to 0.2

RMSE Comparison:

Parameter Name	**Parameter Value**
Number of Antenna Elements	8
Number of Users	31
Amplitude of Sources in volts	1v
Direction of Sources	10:2:70

The direction for the mobiles have been taken in increments of 2 degree starting from 10 to 70.

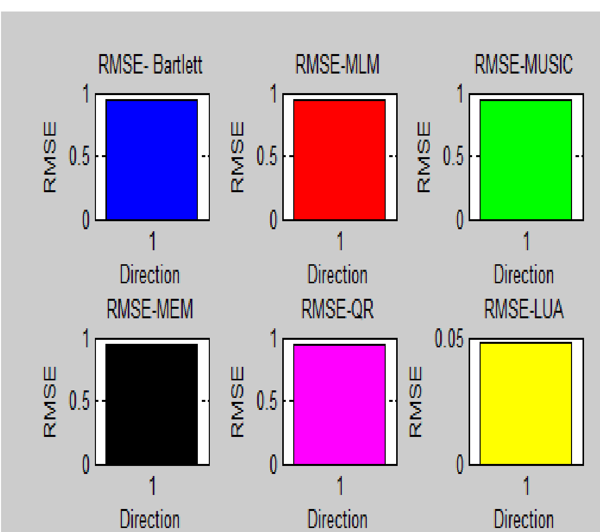

Fig6: RMSE Computation

Fig6 shows the RMSE computation as shown in fig the proposed LUI method has lowest RMSE of 0.05 as compared to remaining methods namely Bartlett, MLM, MEM, MUSIC and QR have RMSE closer to 0.8.

V. CONCLUSION

The various algorithms namely Bartlett, MLM, MEM, MUSIC,QR and LUI algorithms are simulated on various mobile configurations. The following conclusions can be drawn from the results.

1. For the case of Mobile Users which are Far Away and have less RF Sources then LUI,QR MUSIC and MEM performed better and are able to detect the users but Bartlett and MLM method failed to detect
2. For the case of Mobile Users which are Nearby and have less RF Sources then LUI,QR, MUSIC performs better and are able to detect the users but Bartlett, MEM and MLM method failed to detect
3. For the case of Mobile Users which are Far Away and have More RF Sources then all the algorithms perform better
4. For the case of Mobile Users which are Nearby and have More RF Sources then all the algorithms behave well.
5. The bias of the proposed method is lowest as compared to MEM, MLM, MUSIC, Bartlett, MUSIC and QR method.
6. The RMSE of the proposed method is lowest as compared to MEM, MLM, MUSIC, Bartlett, MUSIC and QR method.

REFERENCES

[1] N.D. Sidiropoulos, R.Bro, and G.B Giannakis, " Parallel factor analysis in sensor array processing," *Signal processing, IEEE Transactions* on vol.48,no.8,pp.2377-2388,2000.

[2] P. Stoica and A. Nehorai, "Performance study of conditional and unconditional direction-of-arrival estimation," IEEE Transactions on Acoustics, Speech and Signal Processing, vol. 38, no. 10, pp. 1783 – 1795, October 1990.

[3] H. L. Van Trees, Optimum Array Processing – Part IV of Detection,Estimation and Modulation Theory. Wiley-Interscience, 2002.

[4] Marot, J.Fossati,C.:Bourennane,S.," Fast subspace-based source localization methods," sensor array and multichannel signal processing Workshop,2008. SAM 2008. 5th IEEE,vol.,no.,pp.203,206,21-23 july 2008.

[5] D. T. Vu, A. Renaux, R. Boyer, and S. Marcos, “Some results on the weiss–weinstein bound for conditional and unconditional signal models in array processing,” Elsevier Signal Processing, vol. 95, no. 0, pp. 126 – 148, 2014.
[6] A. Renaux, P. Forster, P. Larzabal, C. D. Richmond, and A. Nehorai,“A fresh look at the bayesian bounds of the weiss-weinstein family,”Signal Processing, IEEE Transactions on, vol. 56, no. 11, pp. 5334 – 5352, November 2008.
[7] Wing-Kin Ma; Tsung-Han Hsieh; Chong-Yung Chi, ”DOA estimation ofquasi-stationary signals via Khatri-Rao subspace,” Acoustics, Speech and Signal Processing, 2009. ICASSP 2009. IEEE International Conference on , vol., no., pp.2165,2168, 19-24 April 2009.

[8] P. Stoica and B. Ng, “On the cramer-rao bound under parametric constraints,” IEEE Signal Processing Letters, vol. 5, no. 7, pp. 177 – 179, July 1998.
[9] T. J. Moore Jr., “A theory of cram´er-rao bounds for constrained parametric model,” Ph.D. dissertation, University of Maryland, College Park,Department of Mathematics, College Park, Maryland, USA, 2010.
[10] F. R¨omer and M. Haardt, “Deterministic cram´er-rao bounds for strict sense non-circular sources,” in International ITG/IEEE Workshop on Smart Antennas (WSA), February 2007.
[11] Y.-H. Li and P.-C. Yeh, “An interpretation of the moore-penrose generalized inverse of a singular fisher information matrix,” IEEE Transactions on Signal Processing, vol. 60, no. 10, pp. 5532 – 5536, October 2012.
[12] P. Laxmikanth, Mr. L. Surendra, Dr. D. Venkata Ratnam, S. Susrutha babu, Suparshya babu “ Enhancing the performance of AOA estimation in wireless communication using the MUSIC algorithm” SPACES-2015, Dept of ECE, K L UNIVERSITY
[13] D. Schulz and R. S. Thom¨a, “Search-based MUSIC techniques for2D DoA estimation using EADF and real antenna arrays,” in 17th International ITG Workshop on Smart Antennas 2013 (WSA 2013), Stuttgart, Germany, 03 2013.
[14] Schmidt, R.O., ”Multiple emitter location and signal parameter estimation,”Antennas and Propagation, IEEE Transactions on , vol.34, no.3, pp.276,280, Mar 1986.
[15] M. Landmann, “Limitations of experimental channel characterisation,”Ph.D. dissertation, Ilmenau University of Technology, Electronic Measurement
Research Laboratory, Ilmenau, Germany, 2007.
[16] M. Landmann, M. K¨aske, and R. Thom¨a, “Impact of incomplete and inaccurate data models on high resolution parameter estimation in multidimensional channel sounding,” IEEE Transactions on Antennas and Propagation, vol. 60, no. 2, pp. 557 – 573, February 2012.
[17] M. Landmann, A. Richter, and R. Thom¨a, “DoA resolution limits in MIMO channel sounding,” in IEEE Antennas and Propagation Society International Symposium, vol. 2, June 2004, pp. 1708 – 1711.
[18] Y. Tian and Y. Takane, “More on generalized inverses of partitioned matrices with banachiewicz–schur forms,” Linear Algebra and its Applications,
vol. 7430, no. 5–6, pp. 1641 – 1655, 2009.
[19] Foutz, Jeffrey, Andreas Spanias, and Mahesh K. Banavar. ”Narrowbanddirection of arrival estimation for antenna arrays.” Synthesis Lectures on Antennas 3.1 (2008): 1-76.